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THE DISPLACEMENT IN ORIGIN FOR THE THERMAL TURBULENT BOUNDARY LAYER

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Abstract. *The present work discusses the dynamic and thermal behaviour of flows that develop over surfaces that present a sudden change in surface roughness and temperature. The theory uses the concept of the displacement in origin to propose a new expression for the logarithmic region of the thermal turbulent boundary layer that leads to an algebraic equation for the estimation of Stanton number. The new expression is of universal applicability, being independent of the type of rough surface – if of the types "K" or "D" – considered. The present formulation is shown to be appropriate for use as wall boundary conditions in two-equation differential formulations of the problem. Experiments were also carried out to validade the theory; measurements of mean velocity and of mean temperature are presented. A reduction of the data provides an estimate of the skin-friction coefficient, of the Stanton number, of the displacement in origin for both the velocity and the temperature profiles. All these parameters were calculated based on the chart method of Perry and Joubert(1963) and on a balance of the integral momentum equation. The paper compares the present data with the theory, showing how any analogy can be drawn between the velocity and the temperature fields.*

Key words: *turbulence, thermal boundary layer, roughness, error in origin.*

1. INTRODUCTION

Parameters of major engineering importance, the calculation of skin-friction coefficient and of Stanton number has always been a very difficult task for fluid flowing over rough surfaces. Some of the methods used to estimate C_f are the momentum-integral equation, pressure-tapped roughness elements and a drag balance. The first method relies on the determination of the derivatives of various mean flow parameters, a process which is known to be highly inaccurate. In addition, this method suffers from any existing threedimensionality in the flow. The fitting of pressure taps in roughness elements is only possible if they are sufficiently large; a condition not normally observed. The drag-balance method needs considerable care to minimize gap leakage; nevertheless, it provides reasonable results. The estimate of S_t, on the other hand, is normally accomplished by evoking Reynolds analogy arguments, which lead to $C_f/2 = S_t$.

On rough wall, methods that resort to the gradient of the log-law cannot be directly used because the effective origin of the boundary layer is not known a priori.

The purpose of this work is to investigate, for a given flow, the behaviour of the error in origin for the velocity and the temperature fields, establishing any existing analogy between them. In order to achieve that, the present work will investigate experimentally the characteristics of turbulent boundary layers that are subjected to a step changes in surface roughness and temperature, with emphasis on the characterisation of the inner layer velocity and temperature profiles.

Because the heat on the wall will be applied just over the rough surface, the properties of the internal layers for velocity and temperature boundary layers will have a different state of development. Parameters of major concern will be the local skin-friction coefficient and Stanton number, and the error in origin for the velocity and temperature fields.

2. SHORT REVIEW

In the past, several studies on the behaviour of boundary layers having a non-uniform distribution of temperature or heat flux at the wall were carried out. For flows over smooth walls, the works of Hartnett(1954), Jonhson(1957, 1959), Reynolds(1958a, 1958b) and Spalding(1964) are classical. Jonhson(1957) reports that for a thermal boundary layer with 4.27 m of unheated starting length and a free stream velocity of 7.62 m/s, measurements taken 1.83 m downstream of the step point reveal that the normalized temperature profiles have shapes different from the normalized velocity profile. The same author, Jonhson(1959), also showed that the temperature intermittency profile has a different form compared with the velocity intermittency profile. Antonia et al.(1977) considered 1.83 m of unheated length, after which a constant surface heat flux was applied. He observed that after 1.8 m of development the temperature profiles had not reached a fully developed form.

For flows over rough surfaces, the number of works is much smaller. The Heat and Mass Transfer Group at Stanford University has been very active for the last three decades, having published a number of reference works. Studies on flows over rough surfaces with changes in the thermal boundary conditions were made by Coleman et al.(1976) and by Ligrani et al.(1979, 1983, 1985). With the help of a kernel function and the superposition of a heat transfer theory, expressions were advanced for the evaluation of Stanton number which were supposed to hold for such different conditions as variable wall temperature, wall blowing and free-stream velocity, and steps in wall temperature and blowing.

Recently, one of the present authors has advanced a theory which is based on an analogy between the transfer of momentum and heat in the inner layers of the boundary layer. The theory considers that the error in origin for both the velocity and the temperature fields have the same order of magnitude to develop expressions for the prediction of C_f and S_t based on two independent algebraic equations. This theory has been formely presented in Silva Freire and Hirata(1990) and in Avelino et al(1998, 1999).

3. EXPERIMENTAL APPARATUS AND PROCEDURE

The experiments were carried out in the high-turbulence wind tunnel sited at the Laboratory of Turbulence Mechanics of PEM/COPPE/UFRJ. The tunnel is an open circuit tunnel with a test section of dimensions 67cmx67cmx3m. The test section is divided into three sections of equal length which can be fitted with surfaces having different types of roughness and of wall heating. The first part, which is normally kept at ambient temperature, consists of a smooth glass wall. The second and third parts of the test section are equipped with independent electric heaters.

The flow was subjected to a step change in roughness after travelling the first meter over the glass wall. The rough surface consisted of a transversely grooved surface with rectangular slats of dimensions 5.92mm x 16mm and pitch of 32.24mm. In constructing the surface, extreme care was taken to keep the first roughness element always depressed below the smooth surface, its crest kept aligned with the smooth glass wall surface.

The glass surface was also followed by a step change in temperature. The second third – next one meter – of the test section had its wall temperature raised by 22.2 $\pm$ 1.5 ^oC. The wall temperature was controlled by 15 termocouples, set at five streamwise stations at three spanwise positions. At the last third of the section, a second step variation in temperature was applied to the flow. This time, the wall temperature was raised by 39 $\pm$ 2 ^oC. Because the wind tunnel was a open circuit tunnel, controlling the temperature in final 0.3 meters was very difficult.

The measurements were performed for values of the free-stream velocity of 3.12 m/s; the free stream level of turbulence was about 2%. The stream-wise pressure gradient was closely set to zero by adjusting the roof of the tunnel according to the readings of eight equally spaced pressure taps.

Mean velocity profiles and turbulence intensity levels were obtained using a DANTEC hot-wire system series 56N. The boundary layer probe was of the type 55P15. A Pitot tube, an electronic manometer, and a computer controlled traverse gear were also used. In getting the data, 10,000 samples were considered which yielded a precision of 0.6% in the mean velocity data. The profiles were constructed from about 100 points taken at stations separated at times by 0.2mm. The mean temperature profiles were obtained through a chromel-constantan micro-termocouple mounted on the same traverse gear system used for the hot-wire probe.

4. THEORY

Before considering the experimental data, let us first review the theory of Silva Freire and Hirata(1990) and of Avelino et al.(1998,1999).

For flows over rough surfaces, Moore(1951) has shown that a universal expression can be written for the wall region provided the origin for measuring the velocity profile is set some distance below the crest of the roughness elements. The displacement in origin is normally referred to in literature as the error in origin, ε. A detailed method to determine the displaced origin can be found originally in Perry and Joubert(1963) and more recently in Perry et al.(1987).

Thus, for any kind of rough surface, it is possible to write

$$\frac{u}{u_\tau} = \frac{1}{\varkappa} \ln \frac{(y_T + \varepsilon) u_\tau}{\nu} + A - \frac{\Delta u}{u_\tau}, \qquad (1)$$

where,

$$\frac{\Delta u}{u_\tau} = \frac{1}{\varkappa} \ln \frac{\varepsilon u_\tau}{\nu} + C_i, \tag{2}$$

and $\varkappa = 0.4$, $A = 5.0$, and $C_i, i = K, D$; is a constant characteristic of the roughness.

The above equations, although of a universal character, have the inconvenience of needing two unknown parameters for their definition, the skin-friction velocity, u_τ, and the error in origin, ε. A chief concern of many works on the subject is, hence, to characterise these two parameters.

In fact, the fundamental concepts and ideas on the problem of a fluid flowing over a rough surface were first established by Nikuradse(1933) who investigated the flow in sand-roughened pipes. Nikuradse was capable to establish that, at high Reynolds number, the near wall flow becomes independent of viscosity, being a function of the roughness scale, of the pipe diameter and of Reynolds number. He also found that, for the defect layer, the universal laws apply to the bulk of the flow irrespective of the conditions at the wall. The roughness effects are, therefore, restricted to a thin wall layer.

Thus, considering that Coles's wake hypothesis applies to the outer region of the flow, the law of the wall can be re-written as

$$\frac{u}{u_\tau} = \frac{1}{\varkappa} \ln \frac{(y_T + \varepsilon)u_\tau}{\nu} + A - \frac{\Delta u}{u_\tau} + \frac{\Pi}{\varkappa} W\left(\frac{y}{\delta}\right), \tag{3}$$

where W is a universal function of y/δ and Π is a parameter dependent on the upstream shear stress and pressure distribution.

Equation 3 provides a representation of the velocity field over the whole of the boundary layer.

Substitution of (y, u)=(δ, U_∞) into equation 3 furnishes

$$\frac{U_\infty}{u_\tau} = \frac{1}{\varkappa} \ln \frac{\delta + \varepsilon}{\varepsilon} + A - C_i + \frac{2\Pi}{\varkappa}. \tag{4}$$

This simple algebraic equation furnishes values of C_f $(= 2\, u_\tau^2/U_\infty^2)$ for known values of U_∞, δ and ε.

To extend expressions 1 and 2 to the temperature turbulent boundary layer we will use the theory of Silva Freire and Hirata(1990). Alternatively, we could have used dimensional arguments. The details of the theory will be omitted; here it suffices to say that, from an asymptotic point of view, the important factor in the determination of the flow structure is the correct assessment of the order of magnitude of the fluctuating quantities. Then, analogies between the transfer of momentum and the transfer of heat can be constructed.

For flows over rough surfaces, we have seen that the characteristic length scale for the near wall region must be the displacement in origin. Indeed, in this situation, the viscosity becomes irrelevant for the determination of the inner wall scale because the stress is transmitted by pressure forces in the wakes formed by the crests of the roughness elements. It is also clear that, if the roughness elements penetrate well into the fully turbulent region, then the displaced origin for both the velocity and the temperature profiles will always be located in the overlap fully turbulent region. The similarity in transfer processes for turbulent flows then suggests that

$$\frac{T_w - t}{t_\tau} = \frac{1}{\varkappa_t} \ln P_r \frac{(y_T + \varepsilon_t)u_\tau}{\nu} + B - \frac{\Delta t}{t_\tau}, \tag{5}$$

where,

$$\frac{\Delta t}{t_\tau} = \frac{1}{\varkappa_t} \ln P_r \frac{\varepsilon_t u_\tau}{\nu} + D_i, \tag{6}$$

and $D_i, i = K, D$; is a constant characteristic of the roughness.

Equations 5 and 6 are the law of the wall formulation for flows over rough surfaces with transfer of heat.

To describe the temperature profile in the defect region of the boundary layer, we may consider that Coles's wake hypothesis also holds for the temperature field so that equation 3 may be re-written as

$$\frac{T_w - t}{t_\tau} = \frac{1}{\varkappa_t} \ln P_r \frac{(y_T + \varepsilon_t)u_\tau}{\nu} + B - \frac{\Delta t}{t_\tau} + \frac{\Pi_t}{\varkappa_t} W\left(\frac{y}{\delta_t}\right), \tag{7}$$

where the wake profile Π_t should, in principle, be a function of the enthalpy thickness.

This equation provides a representation for the temperature field which can be allowed to sustain a different state of development from the velocity field. As a result, Stanton number can be evaluated independently from the skin-friction through a particular equation. To find this equation, we substitute (y, t)=(δ_t, T_∞) into equation 7 to get

$$\frac{T_\infty - T_w}{t_\tau} = \frac{1}{\varkappa_t} \ln P_r \frac{(\delta_t + \varepsilon_t)}{\varepsilon_t} + B - D_i + \frac{2\Pi_t}{\varkappa_t}. \tag{8}$$

This algebraic equation can now be used to find Stanton number as a function of T_∞, δ_t and ε.

The purpose of this work is to investigate the validity of equations 5, 7 and 8 through an experimental accessement of C_f, S_t, ε and ε_t for given flow conditions.

EXPERIMENTS

The global parameters characterising the experiments are shown in Table 1. The station position is given in mm, having as origin the point of transition between the smooth and the rough surfaces.

To find the error in origin, the velocity and temperature profiles were plotted in monolog graphs in dimensional coordinates. Next, the normal distance from the wall was incremented in 0.1 mm intervals and a straight line fit was applied to the resulting points. The best fit was chosen by searching for the maximum coefficient of determination, R-squared. Other statistical parameters were also observed, the residual sum of squares and the residual mean square. Normally, a coefficient of determination superior to 0.99 was obtained. The whole process was extremely laborious because excessive care was taken at this stage of the work.

Table 1: FLOW CONDITIONS.

Station	U_∞	δ	δ_t	δ_2	T_w	T_∞
mm	m/s	mm	mm	mm	^oC	^oC
5	3.12	17.5	–	2.23	48.3	22.7
32.5	3.12	19.5	–	1.94	49.1	22.6
61.0	3.12	21.55	17.5	–	49.1	22.4
129.0	3.18	23.5	23.55	2.73	45.0	22.6
289.0	3.12	27.5	26.5	3.99	44.6	22.4
454.0	3.12	28.5	32.0	4.97	45.4	22.3
612.0	3.12	34.5	39.55	5.29	42.6	22.1
812.0	3.10	44.5	44.55	6.59	44.6	22.1
1027.0	3.18	50.0	50.55	7.81	65.1	22.2
1216	3.12	54.5	54.55	7.51	63.3	22.2
1446	3.12	65.5	59.55	8.27	56.5	22.2
1667	3.18	74.5	69.55	9.12	55.4	22.2

The process of determining the error in origin is typically illustrated in Figures 1 and 2. The results are shown in Figure 3.

No results could be found for the first three stations because of the transitional state of the boundary layer. In fact, the authors could not identify any logarithmic region for both the velocity and the temperature fields. The value of ε_t for stations 8 and 9 could not be determined because, due to physical constraints, the temperature profiles could not be measured.

With the determination of ε and ε_t, one can now use the gradient of the log-law to determine C_f and S_t; this can be made through equations 1 and 5. This method, however, does not lend itself to the establishment of predictive expressions that can be of engineering use.

Expressions of this nature normally resort to combinations of the law of the wall with the law of the wake. Infact, the use of logarithmic expressions for the determination of the skin-friction coefficient and of the Stanton number has become very popular over the years because of the robustness of the method. The fact that they require only a relatively good knowledge of parameters U_∞, δ and of ε, for the estimate of C_f and of S_t makes them a powerful correlation.

To apply equations 4 and 8 to the flow under consideration, the following interpolating functions were used. These relations were taken directly from the experimental data.

$$\delta(x) = 0.00113\,(x + 1000)^{1.405}; \qquad \delta_t(x) = 2.479\,x^{0.435}, \tag{9}$$

$$\varepsilon(x) = 0.000560\,x^{1.069}; \qquad \varepsilon_t(x) = 0.211\,x^{0.408}, \tag{10}$$

the origin of x was taken at the point of step change in roughness.

For the calculations, we used the standard values of the constants. These are shown in Table 2.

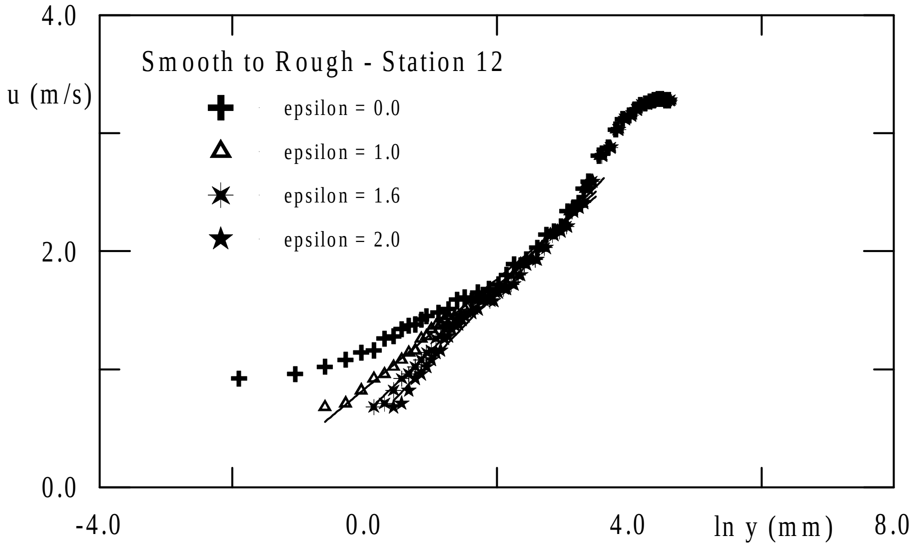

Figure 1. Error in origin, velocity profile.

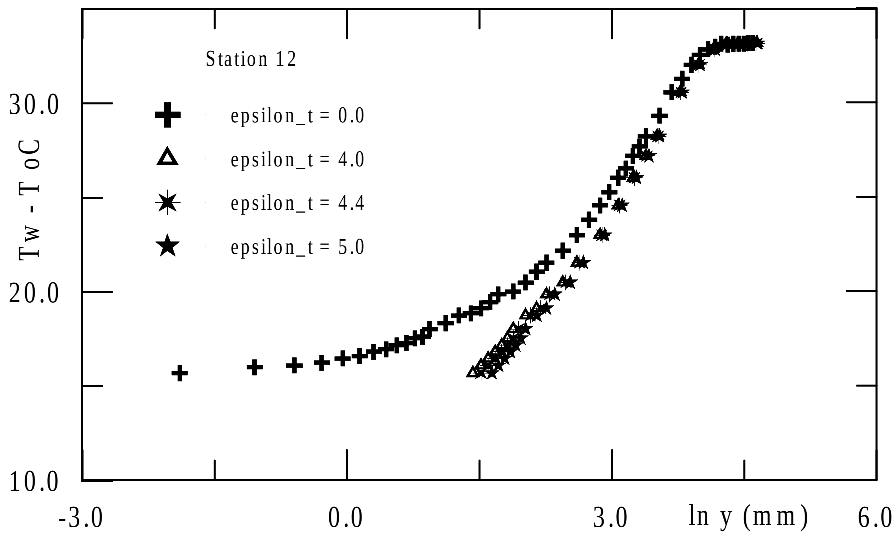

Figure 2. Error in origin, temperature profile.

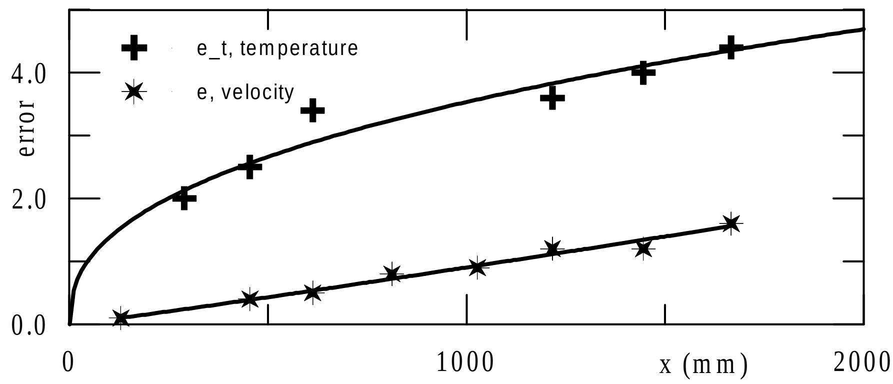

Figure 3. Error in origin.

Table 2: MODEL CONSTANTS.

$\varkappa$	A	C_i	Π	$\varkappa_t$	B	D_i	Π_t
0.4	5.0	3.5	0.55	0.44	5.5	0.5	0.605

Table 3: PREDICTIONS OF SKIN-FRICTION COEFFICIENT AND STANTON NUMBER.

Station mm	C_f Exp.	C_f Eq.(4)	C_f M.I.E.	S_t Exp.	S_t Eq.(8)
5	–	2.82	3.63	–	1.48
32.5	–	2.25	3.67	–	3.45
61.0	–	2.63	3.70	–	3.72
129.0	4.95	3.19	3.78	–	4.09
289.0	3.66	3.89	3.96	4.00	4.50
454.0	4.175	4.29	4.13	3.68	4.72
612.0	4.74	4.55	4.28	4.80	4.85
812.0	2.83	4.76	4.46	–	4.96
1027.0	6.25	4.92	4.63	–	5.03
1216	6.84	5.01	4.78	5.13	5.08
1446	5.09	5.09	4.95	4.87	5.11
1667	4.30	5.15	5.10	4.61	5.14

The friction coefficient and Stanton number obtained through the velocity and temperature gradient method and through the logarithmic equations are shown in Table 4. All values of C_f and of S_t were multiplied by 10^3. The initials M.I.E. stand for momentum integral equation. This approach, as implemented here, only provides qualitative results.

5. FINAL REMARKS

The values of ε and of ε_t were calculated, as said before, according to the method of Perry and Joubert(1973). Systematically adding an arbitrary displacement in origin to the original profiles, the least square method was applied to the near wall points to search for the best straight line fit.

When the authors started the work, they expected, on asymptotic grounds, to find values of ε and of ε_t which would be very close. Unfortunately, this was not the case and it soon became evident that they differ appreciably. We are very sure from the data we collected that, for the present flow situation, Table 2 reflects very well the physics of the phenomenon.

Determining the error in origin has always been a difficult problem that has plagued many authors. Here we have made for, perhaps, the first time in literature, a detailed comparison between ε and ε_t. Since the objective of the work has been to show the usefulness of equations 4 and 8 we have concentrated most of effort in doing this. Some measurements of turbulent quantities and a further processing of the available data will be presented in another opportunity.

Thus, this work has established a working relationship between the rates of growth of the error in origin for the velocity and the temperature profiles. This is a very important matter for it allows Stanton number to be evaluated directly from a proper equation which takes into account the different states of development of the velocity and the temperature boundary layers.

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